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RESEARCH MEMORANDUM

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Abstract

Optimal distortionary taxation is known to display dynamic inconsistency in the sence that an optimal announced tax rate policy gives incentives to renege and, therefore, is not credible in the view of rational consumers, whereas the consistent policy leads to lower social welfare. This paper extends the analysis to contingent tax rate policies, where the government uses information not only on the initial capital endowments but also on the current capital stocks. There are contingent tax rate policies with commitments which yield the first-best result and are, therefore, time-consistent. The credible contingent tax rate policy without commitments may generate more social welfare than the standard optimal tax rate policy and is always better than the consistent tax rate policy with restricted information. In the case of more than one consumer the first-best result can not be achieved anymore. For a growing number of consumers the credible tax rate policy with full information tends to be the same as the consistent tax rate policy with restricted information.

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1. Introduction

Since Kydland and Prescott (1977) have confronted the economics profession with the problem of dynamic inconsistency or time-inconsistency of optimal policies, several authors have treated the problem in a more specific context. The same type of problem occurs for utility maximizers with preferences changing over time and as such the problem goes back to Strotz (1956). The dynamic inconsistency of optimal distortionary taxation has been given considerable attention by Kydland and Prescott (1980), Fischer (1980) and Rogers (1986, 1987). Fischer (1980) uses a stripped-down version of the problem studied by Kydland and Prescott (1980). In this version there are only two periods and one consumer. The consumer takes a savings/consumption decision in the first period and a labour/consumption decision in the second period. The government has to resort to distortionary taxation of both labour and capital income in the second period. Fischer shows that the optimal tax rate policy for the two-period problem is time-inconsistent. That is to say, there is an incentive for the government to renege on the announced policy after the consumer has made the savings/consumption decision in the first period. The reason is that the announcement of taxation of both labour and capital income induces a favourable savings decision, whereas in the second period it is optimal to only tax capital income. The real problem now is that, in the absence of the possibility to externally constrain the behaviour of the government, the announced tax rate policy is not credible and therefore will not be expected by a rational consumer. A requirement of consistency leads to a policy which is credible, that is, which will be believed by a rational consumer. However, such a policy yields lower social welfare. This phenomenon occurs, as Fischer (1980) shows, even if the government maximizes the utility of the (representative) consumer. Rogers (1986) investigates the effects of distributive goals for the model with N consumers. Rogers (1987) extends the model with expenditure taxes and studies the implications.

The issue that seems somewhat overlooked is the role of information. The time-inconsistent optimal tax policy is an "open-loop" policy which only relies on information on the initial capital endowment. Fischer's (1980) consistent tax policy is in fact also an open-loop policy since the announced tax rates are also only a function of the initial capital endowment. It merely has the flavour of a "feedback" policy since reneging on the announced tax rates presupposes knowledge about the capital stock at the beginning of period two.

This paper investigates the impact of information starting from the simple model used by Fischer (1980). It can also be said that this paper extends the analysis from rules to contingent rules. It is shown that if the tax rates may be a function of the capital stock in period one and period two, which will be denoted by "closed-loop" information, it is possible to construct tax rate policies which yield the command optimum. Since the command optimum is reached, these tax rate policies are time-consistent. However, another type of dynamic suboptimality occurs. These policies are only time-consistent on the equilibrium path and not off the equilibrium path, where the equilibrium path is defined as the resulting time path for the capital stock in a behavioural equilibrium between government and consumer. This implies that these policies are not robust against unexpected deviations from the equilibrium path and in that sense they are not credible. A requirement that they are will be called the requirement of strong time-consistency. This resembles the requirement of "subgame perfectness" in an extensive form Nash game (see Selten, 1975). The tax rate policy which is also time-consistent off the equilibrium path will be called the "feedback" policy, because it has the property that it only depends upon information on the current capital stock. It is clear that the requirement of subgame perfectness leads to lower social welfare. It is not immediately clear, however, how this feedback policy compares with the time-consistent and time-inconsistent open-loop policies. The time-inconsistent open-loop policy can be better or worse in terms of social welfare than the feedback policy. However, it is shown that the social welfare resulting from the feedback tax rate policy is higher than the social welfare resulting from the consistent open-loop tax rate policy. It pays to gather information on the capital stock in period two both in terms of robustness against unexpected events and in terms of social welfare. Furthermore, the feedback tax rate policy exists for a wider range of model parameter values. The same results are found for the model with N identical consumers. When the number of consumers goes to infinity, the social welfare gain of the feedback policy over the consistent open-loop policy goes to zero. When the consumers are not identical, that is when they have different initial capital endowments, the closed-loop policy yields the highest social welfare, but not the command optimum.

To summarize, this paper investigates the role of information in a dynamic optimal taxation model. In the case of identical consumers the government can reach the command optimum by using a contingent tax rate policy. When the government can not be committed to stick to an announced tax rate policy and when rational consumers realize this, a tax policy should be credible. For open-loop or restricted information policies credibility means time-consistency. This credible tax rate policy is derived by Fischer (1980) and further exploited by Rogers (1986, 1987). For closed-loop or full information policies credibility means strong time-consistency or subgame perfectness. This credible tax rate policy is derived in this paper and the results are compared with the earlier results.

The organization of the paper is as follows. Section 2 reviews the Fischer analysis and discusses briefly what happens when the consumer realizes the government's budget constraint. Section 3 extends the analysis to closed-loop information and section 4 to N consumers. Section 5 contains some numerical examples and section 6 the conclusions and suggestions for further research.

2. The Fischer analysis

Fischer (1980) considers a two-period taxation problem with only one (representative) consumer. The government taxes the consumer in the second period and spends these taxes on public goods g_2. The consumer has an initial capital endowment $\bar{k}_1$ (>0), consumes c_1 in the first period, consumes c_2 in the second period and works n_2 out of available time $\bar{n}$ in the second period. The production function is linear with marginal productivity of labour equal to a(>0) and marginal productivity of capital equal to b(>0). This implies that the initial capital endowment $\bar{k}_1$ yields $R\bar{k}_1$, with R=1+b, and that the capital stock in the second period k_2 with labour n_2 yields Rk_2+an_2. The government is supposed to maximize the utility of the consumer, which is given by

$$U(c_1,c_2,n_2,g_2) = \ln c_1+\delta\{\ln c_2+\alpha\ln(\bar{n}-n_2)+\beta\ln g_2\} \tag{1}$$

with α,β and $\delta>0$.
The technological constraints are

$$c_1+k_2 = R\bar{k}_1 \tag{2}$$

$$c_2+g_2 = Rk_2+an_2 \tag{3}$$

It is furthermore required that $c_1,c_2,g_2,(\bar{n}-n_2)>0$ and $n_2,k_2\geq 0$.
The highest utility for the government, and thus for the consumer is achieved with

$$c_1^* = \frac{R^2\bar{k}_1+a\bar{n}}{\{1+\delta(1+\alpha+\beta)\}R} \tag{4}$$

$$c_2^* = \delta Rc_1^* \tag{5}$$

$$n_2^* = \bar{n} - \frac{\alpha c_2^*}{a} \tag{6}$$

$$g_2^* = \beta c_2^* \tag{7}$$

and this solution will be called the command optimum.

It is assumed here that $\frac{a\bar{n}}{\delta(1+\alpha+\beta)} < R^2\bar{k}_1 < \frac{\{1+\delta(1+\beta)\}a\bar{n}}{\delta\alpha}$ so that the resulting

$$k_2^* = \frac{\delta(1+\alpha+\beta)R^2\bar{k}_1 - a\bar{n}}{\{1+\delta(1+\alpha+\beta)\}R} \tag{8}$$

and n_2^* are positive, which implies that the maximization problem has an interior solution. Throughout the paper it is assumed that if solutions exist these are interior solutions.

It is clear that the government can reach the command optimum with lump-sum taxation. When the government levies taxes T_2, the optimal reaction of the consumer is

$$c_1 = \frac{R^2\bar{k}_1 + a\bar{n} - T_2}{\{1+\delta(1+\alpha)\}R} \tag{9}$$

$$c_2 = \delta R c_1 \tag{10}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{a} \tag{11}$$

It is easy to show that when the government levies taxes $T_2^*=g_2^*$ the optimal reaction of the consumer is (c_1^*,c_2^*,n_2^*), so that the command optimum results. There is also no dynamic inconsistency. When the government announces taxation T_2^*, the optimal first-period consumption is c_1^* with resulting capital stock k_2^*. Furthermore, the second-period optimal reaction of the consumer to taxation T_2 is

$$c_2 = \frac{Rk_2 + a\bar{n} - T_2}{1+\alpha} \tag{12}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{a} \tag{13}$$

It follows that the second-period optimal taxation is

$$T_2 = \frac{\beta(Rk_2+a\bar{n})}{(1+\alpha+\beta)} \tag{14}$$

Because T_2^* and k_2^* fit equation (14), T_2^* is still the optimal lump-sum taxation after first-period consumption c_1^* led to second-period capital stock k_2^*, so that dynamic inconsistency does not occur. This is of course not surprising, since the command optimum is the best possible result (see also Hillier and Malcomson, 1984), but to spell out the reasoning with equations (12)-(14) might become useful in the sequel.
Fischer (1980) shows that dynamic inconsistency arises when the government has to resort to distortionary taxes t_k on capital income and t_n on labour income. The intuition is clear. It is good to have savings in the first period and to tax only capital income in the second period. In order to induce savings the tax rate on capital income should be relatively low. In the optimum a positive tax rate on labour income results, which becomes suboptimal in the second period. It is important to note that this phenomenon only occurs when the consumer does not take into account that the taxes $t_kRk_2+t_nan_2$ result in government spending g_2. Otherwise, any pair of tax rates (t_k^*,t_n^*) satisfying $g_2^* = t_k^*Rk_2^*+t_n^*an_2^*$ yields the consumers reaction (c_1^*,c_2^*,n_2^*), so that again the command optimum is achieved and dynamic inconsistency disappears. For example, $t_k^* = t_n^* = \frac{\beta}{1+\beta}$ has the desired effect. The Fischer analysis relies on the assumption that the consumer treats government spending as exogenous.
With tax rates t_k and t_n the second-period budget constraint (3) becomes

$$c_2 = (1-t_k)Rk_2+(1-t_n)an_2 \tag{3'}$$

The optimal reaction of the consumer to these tax rates is

$$c_1 = \frac{(1-t_k)R^2\bar{k}_1+(1-t_n)a\bar{n}}{\{1+\delta(1+\alpha)\}(1-t_k)R} \tag{15}$$

$$c_2 = \delta(1-t_k)Rc_1 \tag{16}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{(1-t_n)a} \tag{17}$$

The pair of tax rates (t_k, t_n) that maximizes utility (1), given the technological constraint (2), the behavourial constraints (15)-(17) and the government's budget constraint $g_2 = t_k Rk_2 + t_n an_2$ is given as the solution to the set of equations

$$\frac{t_n \delta\alpha R^2 \bar{k}_1}{(1-t_n)^2} - \frac{t_k a\bar{n}}{(1-t_k)^2} = 0 \tag{18}$$

$$\{\delta(1+\beta) + \frac{\delta\alpha}{1-t_n} + \frac{1}{1-t_k}\}\{(1-t_k)R^2\bar{k}_1 + (1-t_n)a\bar{n}\}$$

$$- \{1+\delta(1+\alpha)\}\{R^2\bar{k}_1 + a\bar{n}\} = 0 \tag{19}$$

The most striking aspect of this result is that $t_n \neq 0$, because this implies that the optimal pair of tax rates is a time-inconsistent tax policy. In the second period, where k_2 is given, the optimal reaction of the consumer to tax rates t_k and t_n is

$$c_2 = \frac{(1-t_k)Rk_2 + (1-t_n)a\bar{n}}{1+\alpha} \tag{20}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{(1-t_n)a} \tag{21}$$

The pair of tax rates (t_k, t_n) that maximizes the second-period part of utility (1), given the capital stock k_2 and the behavioural constraints (20)-(21), is given by

$$t_k = \frac{\beta(Rk_2 + a\bar{n})}{(1+\alpha+\beta)Rk_2} \tag{22}$$

$$t_n = 0 \tag{23}$$

Although they are optimal for the two-period problem, the tax rates that satisfy (18) and (19) are not credible and will not be expected by a rational consumer. In a rational expectations equilibrium equations (22)-(23) have to hold. Fischer (1980) requires consistency of equations (22), (23), (15) and (2) and in this way a consistent tax rate t_k is found as the solution to the quadratic equation[1]

$$\delta(1+\alpha+\beta)R^2\bar{k}_1(1-t_k)^2-\{\delta(1+\alpha)R^2\bar{k}_1+(1-\delta\beta)a\bar{n}\}(1-t_k)+a\bar{n} = 0 \qquad (24)$$

One of the two solutions of (24) typically yields higher social welfare than the other (see appendix). Also, and more importantly, in order to guarantee at least one real solution, the values of the model parameters have to be restricted. Of course, because consistency is an additional restriction, the utility of the consistent tax rate policy is lower than the utility of the optimal tax rate policy.

Fischer (1980) also considers the "inconsistent" solution which results from cheating on (18)-(19) by (22)-(23) under the assumption that the consumer believed the government and consumed c_1 according to (15), (18) and (19). Rogers (1986) investigates for the model with N consumers how the government's distributive goals restrict the problem of time-inconsistency. Rogers (1987) shows that the welfare rankings of expenditure and income taxation may not be preserved under a requirement of time-consistency. This paper will focus mainly on the effects of contingent tax rate policies for the basic model and for the same type of model with N consumers.

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1) It is assumed that $k_2 \neq 0$, which would otherwise lead to a third but uninteresting consistent solution.

3. Contingent tax rate policies

In section 2 the tax policies are a function of time and initial capital endowment. Considering that in this very simple economy the state of the economy is the capital stock, it can also be said that the tax policies are a function of time and initial state. In difference game terminology there is an open-loop information pattern (Başar and Olsder, 1982). The game is of the von Stackelberg type with the government as leader and the consumer as follower. In this terminology the solution $(t_k, t_n, c_1, c_2, n_2)$ of the set of equations (15)-(19) is the open-loop Stackelberg solution of the difference game. Simaan and Cruz (1973a, 1973b) already detected the dynamic inconsistency of this decision model. The consistent tax rate policy from section 2 with the rational reaction of the consumer may be called the consistent open-loop Stackelberg solution (Meijdam and de Zeeuw, 1986). This section extends the analysis to tax rate policies that may also be a function of the current state of the economy or the current capital stock: contingent tax rate policies. In difference game terminology a closed-loop information pattern is considered.

The first step that springs to mind is to reconsider equations (22)-(23) as functional relationships instead of algebraic relationships. The tax rates t_k and t_n are given as a function of the capital stock k_2. This tax rate policy will, by construction, not suffer from dynamic inconsistency. However, there is more. When the tax rules (22) and (23) are substituted into the consumer's reactions (20) and (21) the optimal consumption and labour decisions for period 2, given the capital stock k_2, results:

$$c_2 = \frac{Rk_2 + a\bar{n}}{1+\alpha+\beta} \tag{25}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{a} \tag{26}$$

Equations (22), (23), (25) and (26) form the solution to the optimal taxation problem for the second period given any "initial" capital endowment k_2. It is said that these equations form the behavioural equilibrium of the "subgame" of the second period. On the analogy of a concept in the theory of extensive form Nash games (Selten, 1975) a "subgame perfect" equilibrium will be a solution to the optimal dynamic taxation problem where the behaviour in the second period is given by (22), (23), (25) and (26). Simaan and Cruz (1973a, 1973b) call this a feedback Stackelberg solution and identify it with the idea of dynamic programming. It remains to identify the optimal first period consumption c_1. Straightforward calculations show that

$$c_1 = \frac{R^2\bar{k}_1 + a\bar{n}}{\{1+\delta(1+\alpha)\}R} \tag{27}$$

The requirement of subgame perfectness is stronger than the requirement of time-consistency. Time-consistency requires that (22)-(23) hold for the resulting capital stock k_2. Subgame perfectness requires that (22)-(23) hold for <u>any</u> capital stock k_2. It is also said that subgame perfect policies are not only time-consistent <u>on</u> the equilibrium path but also <u>off</u> the equilibrium path. It means that these tax rate policies are robust against unexpected events that change the capital stock k_2 and in this way they satisfy the strongest idea of credibility. Both the government and the representative consumer make no commitments for the second period. Although the feedback solution is strongly time-consistent, it yields higher utility than the consistent open-loop solution. This is possible because in the feedback solution the government is not restricted to use tax rate policies that are a function of time and initial capital endowment only. The proof of this important result is given in the appendix.

The question arises whether there are contingent tax rate policies with commitments and possibly memory information, that yield a better result in terms of utility than this subgame perfect policy. The answer is yes. There are several contingent tax rate policies that even yield the command optimum. This can be seen as follows. These tax rate policies $(t_k(k_2), t_n(k_2))$ have to induce the consumer to choose (c_1^*, c_2^*, n_2^*) and have to raise total taxation g_2^*. The behaviour of the consumer facing these tax rate policies is given by

$$c_1 = \frac{\{1-t_k(k_2)\}R^2\bar{k}_1 + \{1-t_n(k_2)\}a\bar{n}}{\{1-t_k(k_2)\}R + \delta(1+\alpha)[\{1-t_k(k_2)\}R - t_k'(k_2)Rk_2 - t_n'(k_2)an_2]} \tag{28}$$

$$c_2 = \delta[\{1-t_k(k_2)\}R - t_k'(k_2)Rk_2 - t_n'(k_2)an_2]c_1 \tag{29}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{\{1-t_n(k_2)\}a} \tag{30}$$

where $t_k'(k_2)$ and $t_n'(k_2)$ are the first derivatives of $t_k(k_2)$ and $t_n(k_2)$ with respect to k_2, respectively. Comparing this with the command optimum it follows that $(t_k(k_2), t_n(k_2))$ have to satisfy

$$t_k(k_2^*)R + t_k'(k_2^*)Rk_2^* + t_n'(k_2^*)an_2^* = 0 \tag{31}$$

$$t_k(k_2^*) = \frac{g_2^*}{Rk_2^*} \tag{32}$$

$$t_n(k_2^*) = 0 \tag{33}$$

There are several solutions to (31)-(33). Two examples are the "reciprocal" tax rule

$$t_k(k_2) = \frac{g_2^*}{Rk_2} = \frac{\delta\beta(R^2\bar{k}_1 + a\bar{n})}{\{1+\delta(1+\alpha+\beta)\}Rk_2} \tag{34}$$

$$t_n(k_2) = 0 \tag{35}$$

and the linear tax rule

$$t_k(k_2) = \frac{-g_2^*}{R(k_2^*)^2}k_2 + \frac{2g_2^*}{Rk_2^*} \tag{36}$$

$$t_n(k_2) = 0 \tag{37}$$

The reciprocal tax rule is in fact disguised lump-sum taxation. It says that the government will only tax capital income but always to the amount g_2^*, whatever the consumer's saving decision in the first period has been. The more the consumer has saved the lower will be the tax rate. As can be seen in equation (34) this tax policy is in fact a function of both the current state k_2 and the initial state $\bar{k}_1$. In difference game terminology it is said that there is a closed-loop memory information pattern (Başar and Olsder, 1982). It is well known that this information pattern can give very good results for the leader, and thus in this case also for the follower.
The tax rate policy (34)-(35) induces first-period consumption c_1^* so that capital stock k_2^* results. After substitution of expression (8) for k_2^* the tax rate on capital income becomes

$$t_k = \frac{\delta\beta(R^2\bar{k}_1 + a\bar{n})}{\delta(1+\alpha+\beta)R^2\bar{k}_1 - a\bar{n}} \tag{38}$$

It is easy to show that this expression equals the right-hand side of equation (22) for k_2 given by the right-hand side of equation (8). This implies, together with $t_n=0$, that the tax rate policy (34)-(35) is time-consistent, which is again not surprising since the command optimum is achieved. However, the tax rate policy (34)-(35) is not subgame perfect. For any $k_2 \neq k_2^*$ it becomes suboptimal. This means that the tax rate policy (34)-(35) is time-consistent on the equilibrium path but not off the equilibrium path.
It is clear that the requirement of strong time-consistency or subgame perfectness leads to a loss of social welfare. It is not immediately clear, however, how the social welfare resulting from this policy relates to the social welfare that results from the time-inconsistent and the time-consistent policies of Fischer (1980). In difference game terminology, it is not clear how the outcomes of the open-loop, consistent open-loop, closed-loop memory and feedback Stackelberg solutions relate. The closed-loop memory equilibrium is the best and leads to the command optimum. This implies that there is a consistent contingent tax

rate policy which yields the best possible result. The consistent open-loop equilibrium is worse than the open-loop one. The feedback equilibrium can be better as well as worse than the open-loop equilibrium (see section 5). It can be proven, however, that the feed-back equilibrium is always better than the consistent open-loop equilibrium (see appendix). This implies that there is a strongly time-consistent contingent tax rate policy with a better performance than the time-consistent tax rate policy introduced by Fischer (1980). It pays to gather information on the capital stock in the second period and to choose the contingent tax rate policy (22)-(23) both in terms of robustness against unexpected events and in terms of social welfare. The social welfare is often even higher than for the time-inconsistent open-loop tax rate policy.

The next section will analyse the problem for the somewhat more realistic case of N consumers.

4. N consumers

This section extends sections 2 and 3 to the case where there are N consumers with identical utility functions of the type (1) and identical labour productivity, but with different initial capital endowments $\bar{k}_1^i$, $i=1,\dots,N$. Social welfare, the objective of the government, is assumed to be the sum of the individual utilities

$$\Sigma_{i=1}^N[\ln c_1^i+\delta\{\ln c_2^i+\alpha\ln(\bar{n}-n_2^i)+\beta\ln g_2\}] \tag{39}$$

The technological constraints are

$$\Sigma_{i=1}^N c_1^i+k_2 = R\Sigma_{i=1}^N\bar{k}_1^i \tag{40}$$

$$\Sigma_{i=1}^N c_2^i+g_2 = Rk_2+a\Sigma_{i=1}^N n_2^i \tag{41}$$

where $k_2=\Sigma_{i=1}^N k_2^i$. The command optimum is given by

$$c_1^{i*} = c_1^* = \frac{R^2\bar{k}_1+a\bar{n}}{\{1+\delta(1+\alpha+\beta)\}R} \qquad (i=1,\dots,N) \tag{42}$$

$$c_2^{i*} = c_2^* = \delta Rc_1^* \qquad (i=1,\dots,N) \tag{43}$$

$$n_2^{i*} = n_2^* = \bar{n} - \frac{\alpha c_2^*}{a} \qquad (i=1,\dots,N) \tag{44}$$

$$g_2^* = \beta\Sigma_{i=1}^N c_2^{i*} = \beta Nc_2^* \tag{45}$$

where $\bar{k}_1 = \frac{1}{N}\Sigma_{i=1}^N\bar{k}_1^i$. In the command optimum all the consumers work and consume the same. It is as if they all employ the average initial capital endowment $\bar{k}_1$.

What can the government achieve with lump-sum taxation? That depends. If the government can tax each consumer with a different amount the command optimum can be induced. Each consumer reacts to lump-sum taxation according to equations (9)-(11). It follows, that the optimal lump-sum taxes are

$$T_2^{i*} = R^2\bar{k}_1^i - \frac{\{1+\delta(1+\alpha)\}R^2\bar{k}_1 - \delta\beta a\bar{n}}{\{1+\delta(1+\alpha+\beta)\}} \qquad (i=1,\ldots,N) \tag{46}$$

This tax policy has a strong distributive effect. The higher the initial capital stock the more tax will be levied and for low initial capital endowments the tax may become negative or, to put it differently, may become a subsidy. Each consumer ends up with the same utility regardless of the initial capital endowment.

If the government can only tax each consumer with the same amount the command optimum can not be induced (unless of course each consumer has the same capital endowment). In this case the optimal lump-sum tax per consumer is the solution T_2^* of the equation

$$\Sigma_{i=1}^{N} \frac{1+\delta(1+\alpha)}{R^2\bar{k}_1^i + a\bar{n} - T_2^*} - N\frac{\delta\beta}{T_2^*} = 0 \tag{47}$$

which leads to second-period capital stocks

$$k_2^{i*} = \frac{\delta(1+\alpha)R^2\bar{k}_1^i - a\bar{n} + T_2^*}{\{1+\delta(1+\alpha)\}R} \tag{48}$$

This tax policy does not have such a strong distributive effect. The higher the initial capital endowment the more utility will result. It is important to note that both lump-sum taxation policies require information on the whole set of initial capital endowments or, to put it differently, on the initial state of the economy. Furthermore, both lump-sum taxes are time-consistent. For policy (46) this is again not surprising, since it leads to the command optimum. For policy (47) some analysis is needed. In the second period each consumer reacts to lump-sum taxation according to equations (12)-(13). It follows, that the second-period optimal lump-sum tax per consumer is the solution T_2 of the equation

$$\Sigma_{i=1}^{N} \frac{1+\alpha}{Rk_2^i + a\bar{n} - T_2} - N\frac{\beta}{T_2} = 0 \tag{49}$$

Because T_2^* and k_2^{i*} $(i=1,\dots,N)$ fit equation (49), T_2^* is still the optimal lump-sum taxation per consumer after second-period capital stocks k_2^{i*} have been realized, so that there is no dynamic inconsistency.

What can the government achieve with fixed tax rates? Each consumer reacts to fixed tax rates t_k and t_n according to equations (15)-(17). The pair of tax rates (t_k, t_n) that maximizes social welfare (39), given the technological constraint (2) and the behavioural constraints (15)-(17) for each consumer, and the government's budget constraint

$$g_2 = \Sigma_{i=1}^{N} t_k R k_2^i + t_n a n_2^i \quad , \tag{50}$$

is given as the solution to the set of two equations consisting of equation (19) where $\bar{k}_1$ now is to be interpreted as the average initial capital endowment and

$$\Sigma_{i=1}^{N} \frac{\delta(1+\alpha)(1-t_k)R^2\bar{k}_1^i - (1-t_n)a\bar{n}}{(1-t_k)R^2\bar{k}_1^i + (1-t_n)a\bar{n}}$$

$$- N \frac{(\delta + \frac{\alpha\delta}{1-t_n})(1-t_k)R^2\bar{k}_1 - \frac{1}{1-t_k}(1-t_n)a\bar{n}}{(1-t_k)R^2\bar{k}_1 + (1-t_n)a\bar{n}} = 0 \tag{51}$$

Although it is very difficult to solve these equations analytically, it is immediately clear that the command optimum will not result, simply because different initial capital endowments will lead to different consumer behaviour according to equations (15)-(17). Furthermore, when all capital endowments are the same, equation (51) reduces to equation (18) with $\bar{k}_1$ being the average capital stock, and again, as was shown earlier, the command optimum will not be achieved.

When the consumers take into account that the taxes result in government spending according to the budget constraint (50), a very different problem arises because the consumers do not know each other's behaviour. This problem can be dealt with by solving a Nash game between the consumers under a fixed tax rate regime. It is easy to show that different initial capital endowments again lead to different consumer behaviour, so that this possible direction also diverges from the command optimum.

But also when all consumers have the same initial capital endowments, the command optimum will not be achieved. When all other consumers behave according to (42)-(44), consumer i can get a higher utility by saving less than in the command optimum, because he knows that, given the other consumer's reactions, the decrease in government expenditures g_2 is too small to compensate for the increase in his utility in the first period. This is the well known 'free-rider' behaviour. When the number of consumers goes to infinity, the relative influence of each consumer on government spendings goes to zero and in the limit the open-loop equilibrium results.

The optimal fixed tax rate policy determined by equations (51) and (19) is time-inconsistent. This is not so easy to see as with one consumer, because with N consumers time-consistency does not necessarily imply $t_n=0$. The optimal first-period consumption leads to second-period capital stocks

$$k_2^i = \frac{\delta(1+\alpha)(1-t_k)R^2\bar{k}_1^i-(1-t_n)a\bar{n}}{\{1+\delta(1+\alpha)\}(1-t_k)R} \qquad (i=1,\ldots,N) \qquad (52)$$

In the second period each consumer reacts to fixed tax rates according to equations (20)-(21). The pair of tax rates that maximizes the second-period part of social welfare (39), given the capital stocks k_2^i and the appropriate technological and behavioural constraints, is given by the solution to the set of equations

$$\Sigma_{i=1}^{N} \frac{(1+\alpha)(1-t_n)k_2^i}{(1-t_k)Rk_2^i+(1-t_n)a\bar{n}} - N\frac{(1+\alpha-t_n)\bar{k}_2}{(1-t_k)R\bar{k}_2+(1-t_n)a\bar{n}} = 0 \qquad (53)$$

$$\{1+\beta+\frac{\alpha}{1-t_n}\}\{(1-t_k)R\bar{k}_2+(1-t_n)a\bar{n}\} - (1+\alpha)(R\bar{k}_2+a\bar{n}) = 0 \qquad (54)$$

Substitution of equation (52) in equation (54) leads to equation (19) with $\bar{k}_1$ equal to the average initial capital endowment. Substitution of equation (52) in equation (53) leads to

$$\Sigma_{i=1}^{N} \frac{\delta(1+\alpha)(1-t_k)R^2\bar{k}_1^i-(1-t_n)a\bar{n}}{(1-t_k)R^2\bar{k}_1^i+(1-t_n)a\bar{n}}$$

$$-\ N \frac{(1+\alpha-t_n)\{\delta(1+\alpha)(1-t_k)R^2\bar{k}_1-(1-t_n)a\bar{n}\}}{(1+\alpha)(1-t_n)\{(1-t_k)R^2\bar{k}_1+(1-t_n)a\bar{n}\}} = 0 \tag{55}$$

Under the assumption made in section 2 that there are interior solutions, equation (55) implies that $t_n \leq 0$. The open-loop tax rates t_k and t_n are found as the solution of equations (51) and (19). If these tax rates also solve equation (55) the open-loop tax rate policy would be time-consistent. Comparing equations (51) and (55) it follows that $(1-t_k)(1+\alpha-t_n)=(1+\alpha)(1-t_n)$. This implies that both tax rates have the same sign. Because government spending must be positive, the open-loop tax rate policy is time-inconsistent.

What can the government achieve with contingent tax rates? In principle it is possible to distinguish between uniform tax rates and different tax rates per consumer. It will be clear that the second possibility is more powerful and will generally give higher social welfare. In this paper only the first possibility is investigated. The idea of strong time-consistency or subgame perfectness requires to firstly solve the equations (53) and (54) for the tax rates t_k and t_n. These tax rates are then, together with the second-period behavioural equations (20)-(21) per consumer, considered as functions of the second-period state of the economy $(k_2^1,\ldots,k_2^N)$. The set of equations (53)-(54) is very difficult to solve analytically unless all consumers have the same second-period capital stock or $k_2^i=\bar{k}_2$ for $i=1,\ldots,N$. In that case the solution is

$$t_k(k_2^1,\ldots,k_2^N) = \frac{\beta(R\bar{k}_2+a\bar{n})}{(1+\alpha+\beta)R\bar{k}_2} \tag{56}$$

$$t_n(k_2^1,\ldots,k_2^N) = 0 \tag{57}$$

The marginal changes of these contingent tax rates with respect to changes in the second-period capital stocks k_2^i are

$$t_k^i(k_2^1,\ldots,k_2^N) = \frac{\partial t_k(k_2^1,\ldots,k_2^N)}{\partial k_2^i} = \frac{-\beta a\bar{n}}{N(1+\alpha+\beta)R\{\bar{k}_2\}^2} \tag{58}$$

$$t_n^i(k_2^1,\ldots,k_2^N) = \frac{\partial t_n(k_2^1,\ldots,k_2^N)}{\partial k_2^i} = 0 \tag{59}$$

From equations (58) and (59) it follows that, when the number of consumers N goes to infinity, the marginal tax rates go to zero. This implies that the subgame perfect equilibrium tends to the consistent open-loop equilibrium when the number of (identical) consumers grows.
In case the consumers have different second-period capital stocks k_2^i the strongly consistent contingent tax rates are implicitly given by equations (53) and (54). Implicit differentiation of these equations yields a set of equations for the marginal tax rates t_k^i and t_n^i. In order not to complicate matters unnecessarily this set of equations is not given here, although it is used for the numerical experiments to be presented in the next section.
It remains to show how a contingent tax rate policy in general affects the consumers. In order to be able to evaluate the effect of the first-period consumption decision on the tax rates each consumer has to form expectations about the first period consumption of the other consumers. A Nash game between the consumers is assumed here, which implies that these expectations about each other's decisions are correct. The solution to this Nash game is implicitly given by

$$c_1^i = \frac{c_2^i}{\delta\{(1-t_k)R-t_k^i Rk_2^i-t_n^i an_2^i\}} \qquad (i=1,\ldots,N) \tag{60}$$

$$c_2^i = \frac{(1-t_k)Rk_2^i+(1-t_n)a\bar{n}}{1+\alpha} \qquad (i=1,\ldots,N) \tag{61}$$

$$n_2^i = \bar{n} - \frac{\alpha c_2^i}{(1-t_n)a} \qquad (i=1,\ldots,N) \tag{62}$$

$$k_2^i = R\bar{k}_1^i-c_1^i \qquad (i=1,\ldots,N) \tag{63}$$

where t_k, t_n, t_k^i and t_n^i are functions of the state $(k_2^1,\ldots,k_2^N)$. It is interesting to note that the open-loop Nash equilibrium between the consumers, given a general contingent tax rate policy, is also determined by the set of equations (60)-(63). The strongly consistent or subgame perfect equilibrium for the taxation problem with N consumers is given by equations (60)-(63), (53)-(54) and the set of equations for t_k^i and t_n^i which results from implicit differentiation of (53)-(54).
Again the question arises whether there are contingent tax rate policies that yield a better result in terms of utility than this subgame perfect policy. Again the answer is yes. However, for the problem with N consumers it is not generally possible to reach the command optimum. The government wants to maximize social welfare (39) over the set of contingent tax rate policies, given the behavioural constraints (60)-(63) and the government's budget constraint (50). The resulting social welfare will be higher than in the subgame perfect equilibrium, because the policies do not have to be implicitly given by equations (53)-(54).
Papavassilopoulos and Cruz (1979) show a way how to solve this problem. They transform the closed-loop problem into an open-loop problem with an additional set of instruments consisting of the derivatives t_k^i and t_n^i. This open-loop problem can be solved with standard techniques. A solution to the closed-loop problem is given by tax rules that satisfy the resulting optimal values for the tax rates and the derivatives. It is very difficult, however, to find an analytical solution. In case all consumers have the same initial capital endowment $\bar{k}_1$ there are contingent tax rate policies that yield the command optimum. Comparing the behaviour of the consumers in the command optimum, given by equations (42)-(44), and the behaviour of the consumers facing contingent tax rate policies, given by equations (60)-63), it follows that these policies have to satisfy

$$t_k(k_2^*,\ldots,k_2^*)R+t_k^i(k_2^*,\ldots,k_2^*)Rk_2^*+t_n^i(k_2^*,\ldots,k_2^*)an_2^* = 0 \qquad (i=1,\ldots,N) \qquad (64)$$

$$t_k(k_2^*,\ldots,k_2^*) = \frac{g_2^*}{NRk_2^*} \qquad (65)$$

$$t_n(k_2^*,\ldots,k_2^*) = 0 \tag{66}$$

In case the consumers have different initial capital endowments it is not possible to achieve the command optimum. There is no contingent tax rate policy which induces the same decisions for each consumer as is required in the command optimum.

The results show the same pattern as for the model with one consumer in sections 2 and 3. Although the command optimum is only reached for the model with N identical consumers, the closed-loop tax rate policy again gives the highest social welfare. The open-loop tax rate policy is again time-inconsistent. The feedback tax rate policy converges to the consistent open-loop tax rate policy in the model with N identical consumers when the number of consumers goes to infinity. For the model with different consumers this is probably also true, although this remains to be proven. In the next section the properties of the tax rate policies discussed in sections 2, 3 and 4 are illustrated by means of numerical examples.

5. Numerical examples

In the preceding sections the properties of a number of different tax policies are discussed. In this section some numerical experiments are presented to illustrate these properties.

In all experiments the parameters of the utility function (1), the marginal productivity of labour and capital and the maximum amount of labour a consumer can supply are the same. Their values are: α=0.25, β=0.50, δ=0.90, a=1, b=0.50, which implies that R=1.50, and $\bar{n}$=1.0. These are the same values as Fischer (1980) uses.

In all tables below the following variables are presented. The utility U^i in the equilibrium, the consumption in the first period c_1^i and in the second period c_2^i, the labour supply n_2^i and the tax rates on labour and capital income t_n and t_k, respectively. The value of government spending g_2 is not reported but easily calculated as $\beta\Sigma_{i=1}^{N} c_2^i$. The capital stock in the second period k_2^i is also not reported but can be calculated as $R\bar{k}_1^i - c_1^i$.

For the model with one consumer two different values for the initial capital endowment $\bar{k}_1$ are distinguished: in table 1 $\bar{k}_1$=2.0 and in table 2 $\bar{k}_1$=0.50.

The command optimum is the best possible outcome. This outcome will be achieved when the consumer takes account of the government's budget constraint, with lump-sum taxation and with a closed-loop memory tax rate policy. The open-loop tax rate policy gives a better result than the consistent open-loop policy. As was proven in the appendix, the same applies for the feedback policy. In the example of table 2 the consistent open-loop policy does not exist. For the feedback policy and the open-loop policy the welfare ranking is ambiguous. In the example of table 1 the feedback policy gives the higher social welfare, whereas in the example of table 2 the open-loop policy is better.

In tables 3 and 4 there are two consumers. Table 3 gives the results when both consumers have the same initial capital endowments $\bar{k}_1^1=\bar{k}_1^2$=2.0. In table 4 the endowments differ: $\bar{k}_1^1$=1.70 and $\bar{k}_1^2$=2.30. The command optimum is the same in both examples, because it only depends on the average initial endowment. With identical endowments both the closed-loop memory tax rate policy and the uniform lump-sum taxation policy lead to the command optimum. With different endowments the command optimum does not result. The closed-loop tax rate policy is always better then the open-loop and the feedback policies, which in turn are better than the consistent open-loop policy (if it exists). The ranking of the feedback and the open-loop policies is ambiguous again. When the consumers take account of the government's budget constraint the result is always better than when they do not. The command optimum will not be achieved, however, even when both consumers have the same endowments.

Table 1: One consumer, $\bar{k}_1 = 2.0$

	U	c_1	c_2	n_2	t_n	t_k
command optimum	0.759	1.424	1.922	0.519	-.-	-.-
open-loop	0.706	1.726	1.553	0.419	0.332	0.334
feedback	0.724	1.725	1.664	0.584	0.000	0.435
consistent open-loop	0.625	2.014	1.417	0.646	0.000	0.479

Table 2: One consumer, $\bar{k}_1 = 0.5$

	U	c_1	c_2	n_2	t_n	t_k
command optimum	-1.690	0.550	0.743	0.814	-.-	-.-
open-loop	-1.719	0.587	0.653	0.734	0.386	0.176
feedback	-1.725	0.667	0.643	0.839	0.000	2.571
consistent open-loop	does not exist for these parameter values					

Table 3: Two consumers, $\bar{k}_1^1=2.0$, $\bar{k}_1^2=2.0$

	i	U^i	c_1^i	c_2^i	n_2^i	t_n^i	t_k^i	T^i
command optimum	1	1.071	1.424	1.922	0.519	-.-	-.-	-.-
	2	1.071	1.424	1.922	0.519	-.-	-.-	-.-
	tot.	2.142	2.848	3.845	1.039	-.-	-.-	-.-
closed-loop	1	1.071	1.424	1.922	0.519	0.000	0.407	-.-
	2	1.071	1.424	1.922	0.519	0.000	0.407	-.-
	tot.	2.142	2.848	3.845	1.039	-.-	-.-	-.-
uniform lump-sum	1	1.071	1.424	1.922	0.519	-.-	-.-	0.961
taxation	2	1.071	1.424	1.922	0.519	-.-	-.-	0.961
	tot.	2.142	2.848	3.845	1.039	-.-	-.-	1.922
open-loop with	1	1.060	1.561	1.755	0.474	0.331	0.334	-.-
budget restriction	2	1.060	1.561	1.755	0.474	0.331	0.334	-.-
	tot.	2.120	3.122	3.511	0.948	-.-	-.-	-.-
open-loop	1	1.018	1.726	1.553	0.419	0.332	0.334	-.-
	2	1.018	1.726	1.553	0.419	0.332	0.334	-.-
	tot.	2.036	3.453	3.106	0.838	-.-	-.-	-.-
feedback	1	1.005	1.837	1.568	0.608	0.000	0.449	-.-
	2	1.005	1.837	1.568	0.608	0.000	0.449	-.-
	tot.	2.010	3.674	3.137	1.216	-.-	-.-	-.-
consistent	1	0.937	2.014	1.417	0.646	0.000	0.479	-.-
open-loop	2	0.937	2.014	1.417	0.646	0.000	0.479	-.-
	tot.	1.874	4.027	2.834	1.292	-.-	-.-	-.-

Table 4: Two consumers, $\bar{k}_1^1=1.70$, $\bar{k}_1^2=2.30$

	i	U^i	c_1^i	c_2^i	n_2^i	t_n^i	t_k^i	T^i
command optimum	1	1.071	1.424	1.922	0.519	-.-	-.-	-.-
	2	1.071	1.424	1.922	0.519	-.-	-.-	-.-
	tot.	2.142	2.848	3.845	1.039	-.-	-.-	-.-
closed-loop	1	0.848	1.321	1.700	0.601	-0.065	0.425	-.-
	2	1.271	1.524	2.181	0.488	-0.065	0.425	-.-
	tot.	2.118	2.844	3.882	1.089	-.-	-.-	-.-
uniform lump-sum	1	0.730	1.218	1.644	0.589	-.-	-.-	0.944
taxation	2	1.364	1.641	2.216	0.446	-.-	-.-	0.944
	tot.	2.094	2.859	3.518	1.035	-.-	-.-	1.887
open-loop with	1	0.794	1.411	1.525	0.557	0.247	0.353	-.-
budget restriction	2	1.263	1.741	1.992	0.442	0.247	0.353	-.-
	tot.	2.090	3.152	3.518	0.999	-.-	-.-	-.-
open-loop	1	0.741	1.520	1.363	0.497	0.323	0.336	-.-
	2	1.263	1.943	1.743	0.357	0.323	0.336	-.-
	tot.	2.004	3.463	3.106	0.854	-.-	-.-	-.-
feedback	1	0.801	1.676	1.426	0.679	-0.109	0.487	-.-
	2	1.214	1.898	1.843	0.585	-0.109	0.487	-.-
	tot.	2.015	3.574	3.268	1.263	-.-	-.-	-.-
consistent open-loop		does not exist for these parameter values						

6. Conclusions

This paper considers a simple dynamic taxation problem where the government maximizes a utilitarian social welfare function. It is shown that in the case of one (representative) consumer there exist contingent tax rate policies which lead to the command optimum. However, these contingent tax rate policies are not credible in the sense that they are not time consistent off the equilibrium path. A rational consumer will not believe these policies in case the government does not have to make commitments and mistakes or unexpected events are not excluded. A requirement of credibility or strong time-consistency or subgame perfectness leads to a social welfare result which is lower than the command optimum but higher than the time-consistent outcome derived by Fischer (1980). This is possible because here the government can employ information on the state of the economy at the time of the actual decisions. Depending on the parameters of the model this subgame perfect outcome can either be better or worse than the time-inconsistent result derived by Fischer (1980). In the case of more than one consumer generally the same welfare ranking results. With a growing number of consumers the social welfare gain of the subgame perfect equilibrium over the time-consistent equilibrium tends to disappear. The command optimum is only achieved with a contingent tax rate policy when the consumers have the same initial capital endowments.

This paper does not consider the possibility that the government can employ a different tax rate for each consumer. It is clear that in this way the command optimum can be reached. A precise treatment of this possibility is subject of further research.

The model used in this paper is very simple. What happens with other, more general utility functions or technologies? What is the effect of the introduction of expenditure taxes or distributive goals? How does an extension of the time horizon with labour supply and government spending in all periods affect the results in this paper? What are the effects of embedding these contingent tax rate policies in an intergenerational model? All these questions are left for further research.

Appendix

This appendix proves for the model with one consumer that the feedback equilibrium is better than the consistent open-loop equilibrium. As in section 2 it is assumed that all maximization problems have interior solutions.

Given first-period consumption c_1, the optimal values for the other decision variables are

$$c_2 = \frac{R^2\bar{k}_1 + a\bar{n} - Rc_1}{1+\alpha+\beta} \tag{a.1}$$

$$n_2 = \bar{n} - \frac{\alpha c_2}{a} \tag{a.2}$$

$$g_2 = \beta c_2 \tag{a.3}$$

which results in utility

$$\tilde{U}(c_1) = \ln c_1 + \delta(1+\alpha+\beta)\ln(R^2\bar{k}_1 + a\bar{n} - Rc_1) + U_0 \tag{a.4}$$

where U_0 is constant.

The optimal $\tilde{U}$ is reached for the command optimum value c_1^* given by equation (4). To the right of c_1^* the function $\tilde{U}$ is strictly decreasing. The feedback equilibrium value c_1^{FB} is given by equation (27). The values of the other decision variables are given by equations (25), (26), (22), (23) and the government's budget constraint. Equation (25) and the technological constraint (2) yield equation (a.1). Equations (22), (23), (25) and the government's budget constraint yield equation (a.3). It follows that equations (a.1)-(a.3) are satisfied, so that the utility in the feedback equilibrium is given by $\tilde{U}(c_1^{FB})$.

The consistent open-loop equilibrium value c_1^{COL} is given by equation (15), where $t_n=0$ according to equation (23) and t_k has to satisfy the quadratic equation (24). The values of the other decision variables in this equilibrium are given by equations (16), (17), (23), (24) and the government's budget constraint. Equations (15), (16), (22), (23) and the technological constraint (2) yield equation (a.1). Equations (17) and

(23) yield equation (a.2). Equations (22), (23), (a.1), the technological constraint (2) and the government's budget constraint yield equation (a.3). It follows that equations (a.1)-(a.3) are satisfied, so that the utility in the consistent open-loop equilibrium is given by $\tilde{U}(c_1^{COL})$.

Because $\beta>0$ it follows that $c_1^{*}<c_1^{FB}$, so that $\tilde{U}(c_1^{*})>\tilde{U}(c_1^{FB})$, which is not surprising since the command optimum gives the highest utility. More interesting is, that if $t_k>0$ it follows that $c_1^{FB}<c_1^{COL}$ so that $\tilde{U}(c_1^{FB})>\tilde{U}(c_1^{COL})$. This implies that, if there exists a consistent open-loop equilibrium, this equilibrium is worse than the feedback equilibrium. Existence of a consistent open-loop equilibrium is namely based on the existence of real roots to the quadratic equation (24), and if these real roots exist they are positive.

This analysis also shows clearly that the smallest of the two real roots of equation (24), if they exist, gives the highest utility and thus leads to the consistent open-loop equilibrium.

Finally it should be noted that the open-loop equilibrium can not be compared with the other equilibria in this way, because equations (a.1)-(a.3) are not satisfied in this equilibrium.

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